a paper by Michal Kolesár & Mikkel Plagborg-Møller (Princeton)

Dynamic Causal Effects in a Nonlinear World

The Good, The Bad & The Ugly

The Eternal Endogeneity Problem

We are interested in dynamic causal effects of D_t on Y_{t+h}

Macro: suppose $\exists$ "structural shock" X_t

- Distribution of X_t independent of everything else in DGP

Two choices for estimation

- VAR, assumes specific propagation of X_t (extrapolates)

$$\frac{\partial Y_t}{\partial X_t} = B, \frac{\partial Y_{t+1}}{\partial X_t} = AB, \ldots, \frac{\partial Y_{t+h}}{\partial X_t} = A^h B.$$

- Local Projection: estimate dyanamics directly (OLS w/ Y_{t+h} for each h)

Macro Solution

Empirical part of most monetary policy papers

$$\text{Reg } Y_{t+h} \text{ on } f(Z_t) \text{ (and controls)}$$

where Z_t is an off the shelf "monetary policy shock" (MPS)

Used to provide answers to questions like

"How does monetary policy affect y?"

"How does s affect monetary policy's effects on y?"

But can we actually answer these questions?

- OLS is linear, what about nonlinearities?
- We only have Z_t not X_t

Roadmap

Rambachan and Shephard (2025), Kolesár and Plagborg-Møller (2025) ask

what are VARs/LPs estimating?

Under common assumptions about Z_t, **weighted average** of (causal) marginal effects

Reprises micrometrics results from 90's, which we know aren't a free lunch

- Weights can be negative (Small et al., 2017; Goldsmith-Pinkham et al., 2024)
- Weights are hard to interpret (Masten, 2025)

KPM: Macro assumptions make weight estimation easy

KPM Framework

arbitrary DGP $\psi_h : \mathbb{R} \times \mathbb{R}^L \to \mathbb{R}$ for an outcome variable Y at time $t+h$

$$Y_{t+h} = \psi_h(X_t, \boldsymbol{S_{t+h}}) \tag{1}$$

$$\Psi_h(x) \equiv \mathbb{E}[\psi_h(x, \boldsymbol{S_{t+h}})] \tag{2}$$

where $\boldsymbol{S_{t+h}}$ is “everything else”. Consider a local projection

$$Y_{t+h} = \alpha + \beta X_t + \gamma' \boldsymbol{W_t} + e_{t+h}$$

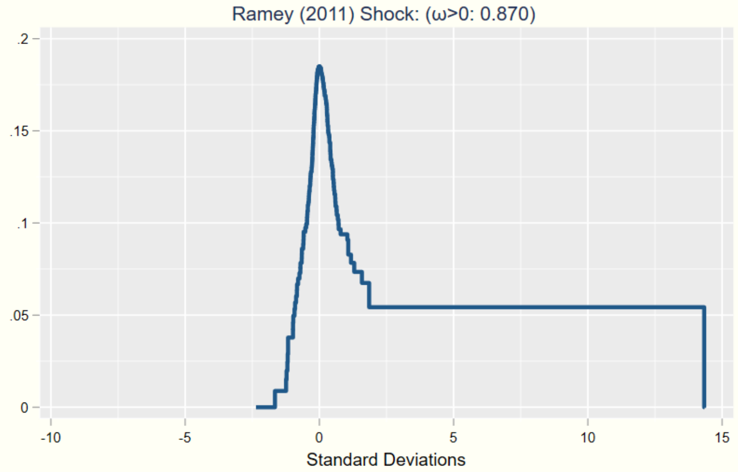
KPM Proposition 1: if X_t is a structural shock, $\Psi_h(x) = \mathbb{E}[\psi_h(X_t, \boldsymbol{S_{t+h}}) \mid X_t = x] \equiv m_h(x)$

$$\beta = \int_I \omega(x) \cdot \Psi_h'(x)\,\mathrm{d}x \tag{3}$$

$$\text{with} \;\; \omega(x) = \frac{\mathrm{Cov}(\mathbf{1}_{\{x \leq X_t\}}, X_t)}{\mathrm{Var}(X_t)} \geq 0 \tag{4}$$

Plotting Weights

Let X_t be the government spending shock from Ramey (2011)



(Regression of $\mathbb{1}(x \leq X_t)$ on X_t)

Processing (1)

Another local projection

$$Y_{t+h} = \alpha + \beta_1 f_1(X_t) + \cdots + \beta_N f_N(X_t) + \gamma' \mathbf{W_t} + e_{t+h}$$

Corollary 1: For $X_i^{\perp}$ is projection residuals of f_i on $\{f_j\}_{j \neq i}$

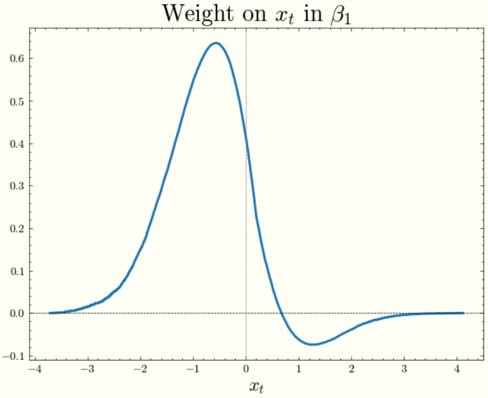
$$\beta_i = \int_I \omega_i(x) \cdot \Psi'_h(x)\,\mathrm{d}x \qquad \text{with } \omega_i(x) = \frac{\mathrm{Cov}(\mathbf{1}_{\{x \leq X_t\}}, X_i^{\perp})}{\mathrm{Var}(X_i^{\perp})}$$

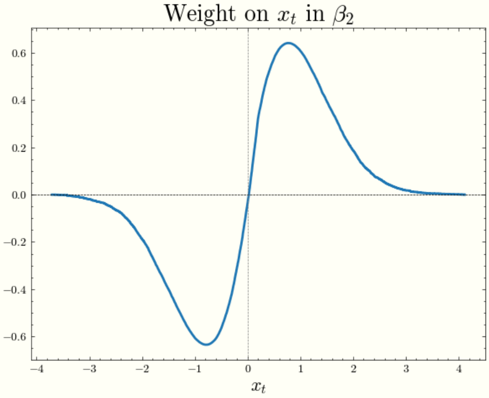
Corollary 2: For $N = 2$ and $f_1 = X_t$, $\int \omega_2(x)\,\mathrm{d}x = 0 \implies \omega_2(\cdot) \ngeq 0$

- White (1980)

Example (interactions)

$$Y_{t+h} = \alpha + \beta_1 X_t + \beta_2 \max\{X, 0\} + e_{t+h}$$





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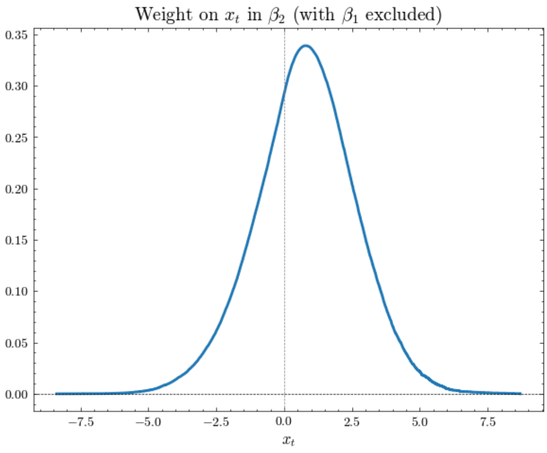
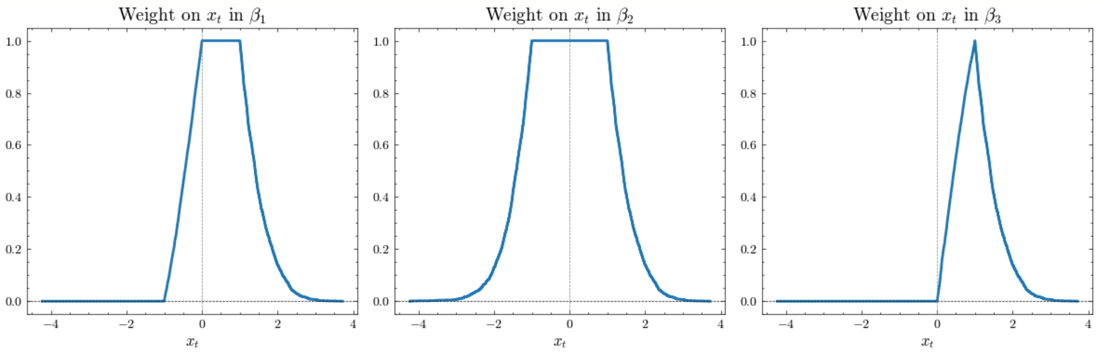


Example (dummies)

Partition $\mathbb{R}$ into 4 regions, exclude $x > 1$

$$Y_{t+h} = \alpha + \beta_1 \cdot \mathbf{1}_{-x_t \in (0,1]} - \beta_2 \cdot \mathbf{1}_{-x_t < 1} - \beta_3 \cdot \mathbf{1}_{x_t \in [0,1]} + e_{t+h}$$



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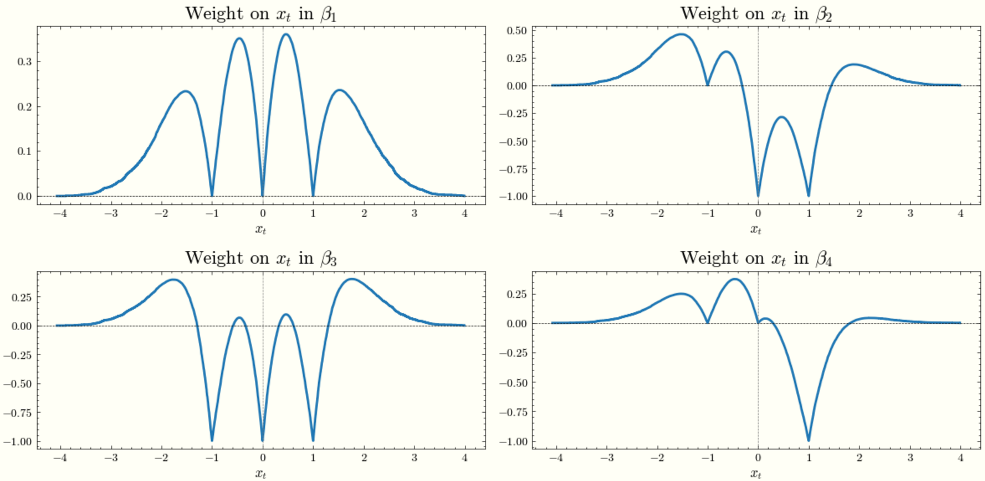


Processing (2)

Other implications:

- weights don't depend on outcome
- Proposition 3: weights still depend on X_t even when we use proxy Z_t

$$\frac{\mathrm{Cov}(\mathbf{1}_{\{x \leq X_t\}}, \zeta(X_t))}{\mathrm{Var}(Z_t)} \quad \text{with} \;\; \zeta(x) = \mathbb{E}[Z_t \mid X_t = x]$$

- what about finite sample properties?
- still just a "summary statistic" (black box)

Unpacking the Weights

$$Y_{t+h} = \alpha + \beta_1 f_1(X_t) + \cdots + \beta_N f_N(X_t) + \gamma' \mathbf{W_t} + e_{t+h}$$

Note: $X_i^\perp$ is mean 0 $\implies \omega_i(-\infty) = 0$.

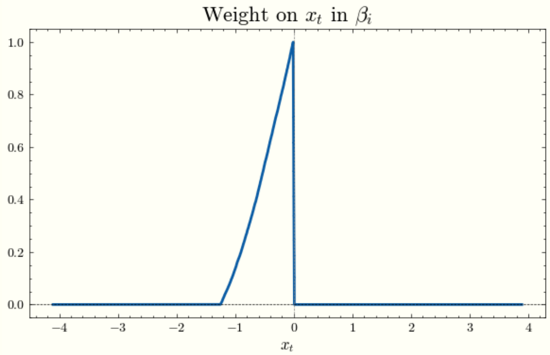
$$\omega_i(x) \propto \text{Cov}(\mathbf{1}_{X_t \geq x}, X_i^\perp) = \int_x^\infty X_i^\perp(a) \cdot f(a) \, \mathrm{d}a$$

Weight is balance between remaining probability weighted mass of + vs. - $X_i^\perp$

Unpacking the Weights (Example)

Fix $\delta > 0$ small. Let $I = [-1, \delta)$, $I^\delta = [-\delta, \delta]$ and $p = \mathbb{P}(X_T \in I)$, $p_\delta = \mathbb{P}(X_T \in I^\delta)$

$$X_i^\perp(a) = \begin{cases} -\left[1 - \frac{p}{p+p_\delta}\right] & a \in I \\ \frac{p}{p+p_\delta} & a \in I^\delta \\ 0 & \text{o.w} \end{cases}$$



What if X_t is not a structural shock?

$$Y_{t+h} = \alpha + \beta X_t + \gamma' \mathbf{W_t} + e_{t+h}$$

$$\beta = \iint \omega(x, \mathbf{w}) m_h'(x, \mathbf{w}) \, \mathrm{d}x \, \mathrm{d}\mathbf{w}$$

KPM Proposition 7: if $\mathbb{E}[X_t \mid \mathbf{W}_t]$ not linear in $\mathbf{W}_t$, $\omega(\cdot) \not\geq 0$

General Statement: conditional mean must be in class Γ your regression allows

Intuition: let $\pi(\cdot)$ be the best approximation in Γ of true CMF π^*

- As $x \to -\infty$, ω will have same sign as $\pi^* - \pi$
- $\pi^* - \pi$ will average to zero, so must have some negative weight

Conclusion

How much should we worry about negative weight?

Kitagawa et al. comment: positive weight form might still exist

- Let $m_h'(a) = m_h'(b)$ w/ $\omega(a) < 0$. Redefine $\tilde{\omega}(a) = 0$, $\tilde{\omega}(b)$ absorbs leftover weight
- "more complier than defiers" (de Chaisemartin, 2017)

OLS is still a black box, but KPM help reduce opacity

- Straying from baseline requires care

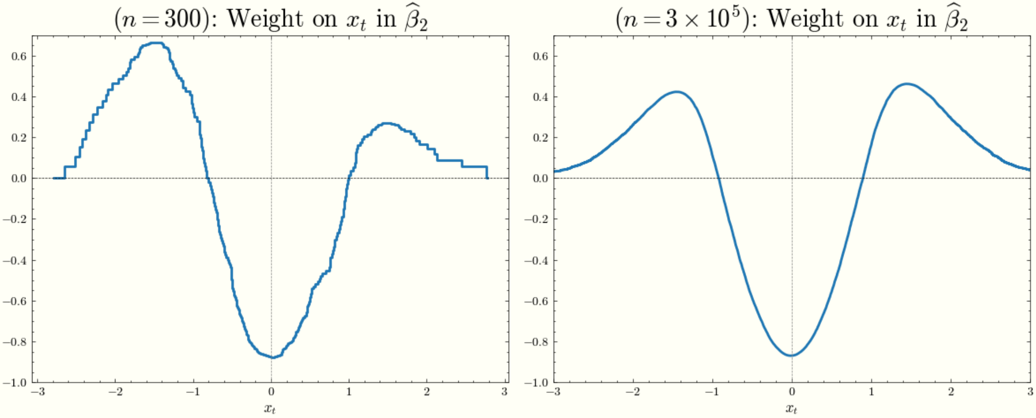
Recap

$$\beta = \int \omega(a) g'(a) \mathrm{d}a$$

- We can estimate $\hat{\beta}$, a weighted average of an object of interest $g'(\cdot)$
- We can estimate the weighting scheme $\hat{\omega}(\cdot)$
- .. but this is still a black box
- is it possible to manipulate the regression to better characterize $g'(\cdot)$?

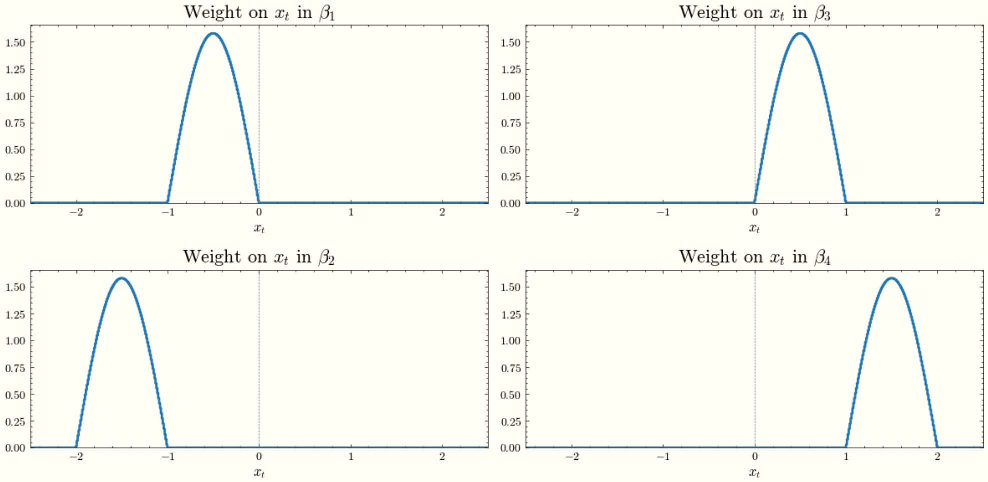
Finite Sample Properties

$$Y_{t+h} = \alpha + \beta_1 X_t + \beta_2 X_t^3 + \gamma' \mathbf{W_t} + e_{t+h}$$



Slicing up Support

Re: partitioning. It would be nice if



Indicator Functions Work Well

$$Y_{t+h} = \alpha + \beta_1 f_1(X_t) + \beta_2 f_2(X_t) + \beta_3 f_3(X_t) + \beta_4 f_4(X_t) + \gamma' \mathbf{W_t} + e_{t+h}$$