Uncovering Nonlinearities with Regression Anatomy

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The (Revived) Power of Least Squares

1990's: Regression estimands are weighted average of marginal effects

- Imbens and Angrist (1994), Yitzhaki (1996), Angrist et al. (2000)

..but not a free lunch

- Weights can be negative (Small et al., 2017; Goldsmith-Pinkham et al., 2024)
- Weights are hard to interpret (Masten, 2025)

Macro: Rambachan and Shephard (2025), Kolesár and Plagborg-Møller (2025) ask

what are VARs/LPs estimating?

- Under common assumptions about shock series, **recover powerful result**

Roadmap

Taking this framework farther $\implies$ clear procedure to think about nonlinearities

Today: focus on

1. New perspective of VARs/LPs
2. Implementation

Application: U.S. monetary policy shocks

- find lots of evidence of nonlinearities
- hard to match with workhorse non-linear DSGE

Regression Weights

$$Y_{t+h} = \alpha + \beta X_t + \gamma' \boldsymbol{W_t} + e_{t+h}$$

Suppose X_t is a shock (independent of everything else in DGP)

Then β is a weighted average of X_t's marginal effect on Y_{t+h}

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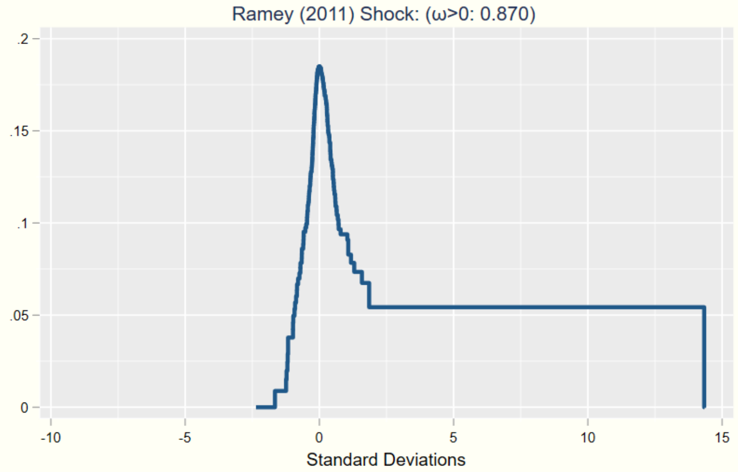
- weights $\omega(a)$ defined for each a in the support of X_t.

$$\omega(a) = \frac{\text{Cov}(\mathbb{1}(a \leq X_t), X_t)}{\text{Var}(X_t)}$$

- K&P-M: just one regression for each a to estimate weights

Plotting Weights

Let X_t be the government spending shock from Ramey (2011)



(Regression of $\mathbb{1}(a \leq X_t)$ on X_t)

Recap

$$\beta = \int \omega(a) g'(a) \mathrm{d}a$$

- We can estimate $\hat{\beta}$, a weighted average of an object of interest $g'(\cdot)$
- We can estimate the weighting scheme $\hat{\omega}(\cdot)$
- .. but this is still a black box
- is it possible to manipulate the regression to better characterize $g'(\cdot)$?

Thinking about Nonlinearities

Example: suppose X_t is a monetary policy shock

- Are the (absolute) effects of expansionary and contractionary shocks the same?

Would be nice if we could find f_1, f_2 such that in

$$Y_{t+h} = \alpha + \beta_1 f_1(X_t) + \beta_2 f_2(X_t) + \gamma' \boldsymbol{W_t} + e_{t+h}$$

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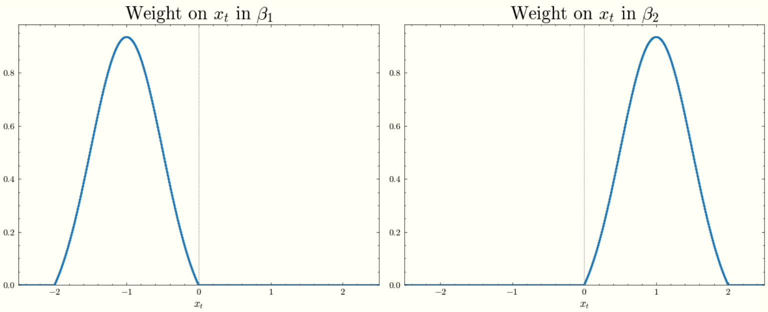
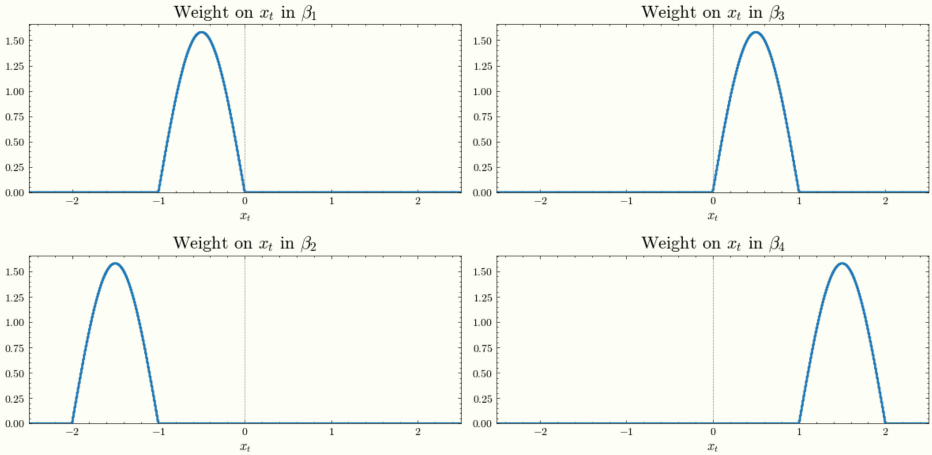


Generalizing

We can push this even further:

$$Y_{t+h} = \alpha + \beta_1 f_1(X_t) + \beta_2 f_2(X_t) + \beta_3 f_3(X_t) + \beta_4 f_4(X_t) + \gamma' \mathbf{W_t} + e_{t+h}$$



Overview

Yes it is possible

- ..but hard to find anything so tidy with small sample size
- **Main recommendation**: specific indicator functions work surprisingly well

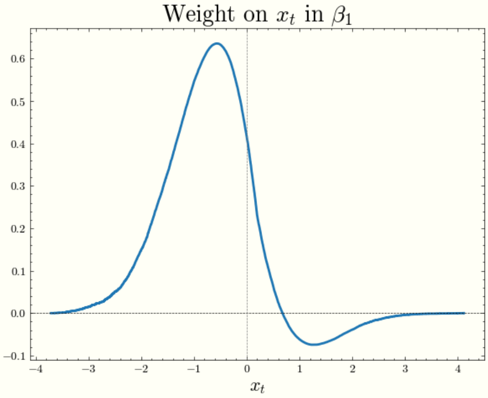
But first: why is usual procedure insufficient?

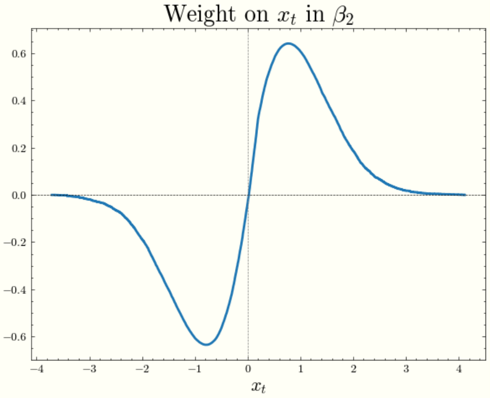
- Also, words of caution for new procedure

Sign Effects – Caution in Interpretation

Suppose we are just interested in cuts and hikes (e.g., Alessandri et al., 2025)

$$Y_{t+h} = \alpha + \beta_1 X_t + \beta_2 X_t^+ + \gamma' \boldsymbol{W_t} + e_{t+h}$$

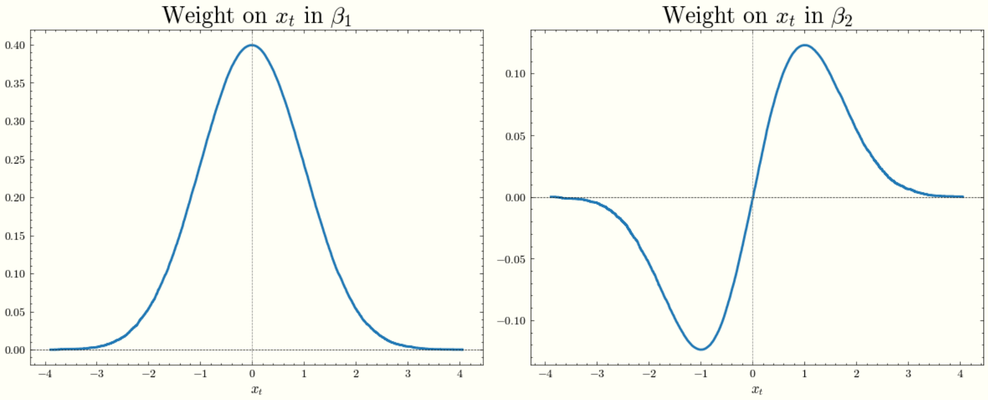




Sign Effects – Symmetric Case Can Work Well

Alternative for symmetric shocks:

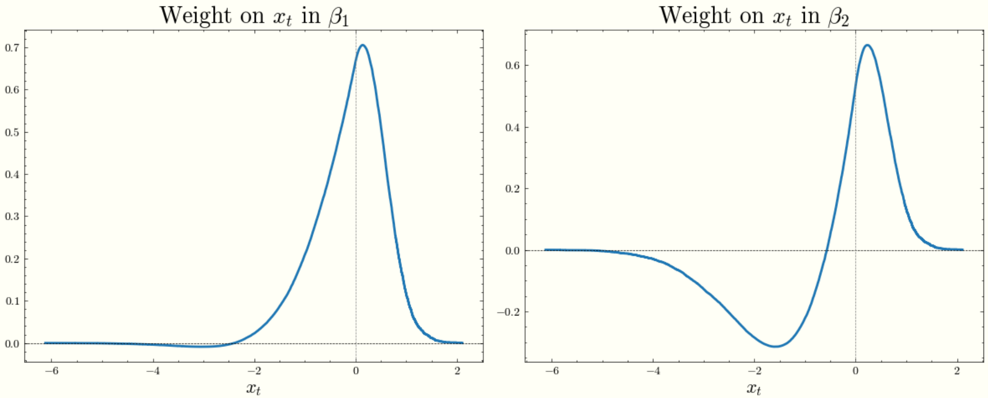
$$Y_{t+h} = \alpha + \beta_1 X_t + \beta_2 |X_t| + \gamma' \mathbf{W_t} + e_{t+h}$$



Note: this also works for X_t^2 (Caravello and Martínez Bruera, 2024)

What if Shock is Asymmetric?

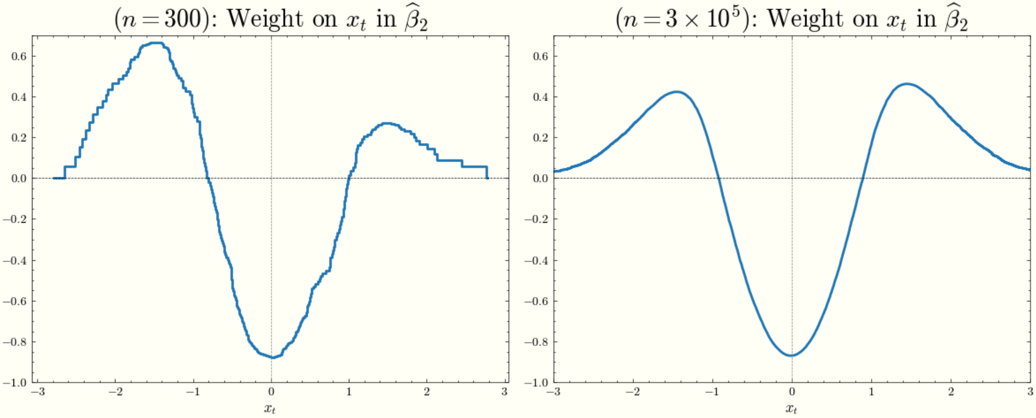
$$Y_{t+h} = \alpha + \beta_1 X_t + \beta_2 |X_t| + \gamma' \mathbf{W_t} + e_{t+h}$$



Note: if $|\omega(a)| \neq |\omega(-a)|$, inference can be distorted (C & M B, 2024)

Only Size Effects, Symmetric Shock

$$Y_{t+h} = \alpha + \beta_1 X_t + \beta_2 X_t^3 + \gamma' W_t + e_{t+h}$$



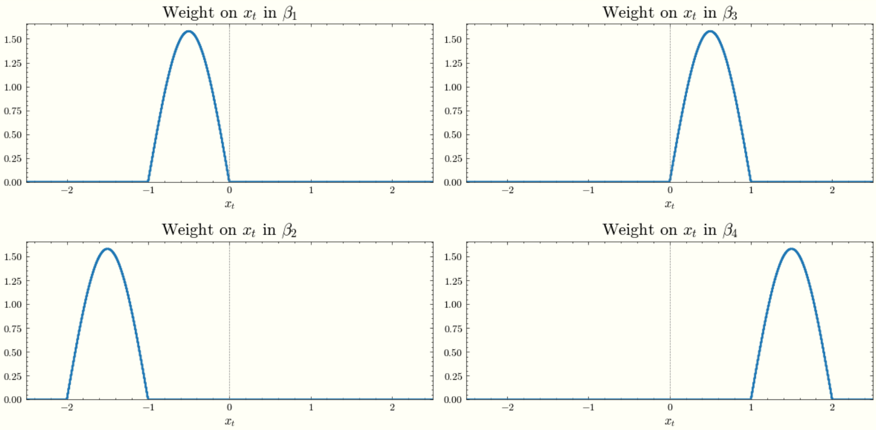
Takeaways

1. Weighting not obvious ex ante (always check)
2. Including polynomial terms not sufficient (White, 1980)
3. Easy to conflate size and sign effects (Caravello and Martínez Bruera, 2024)
4. Traditional approach is sensitive to shock distribution and convergence

Solution

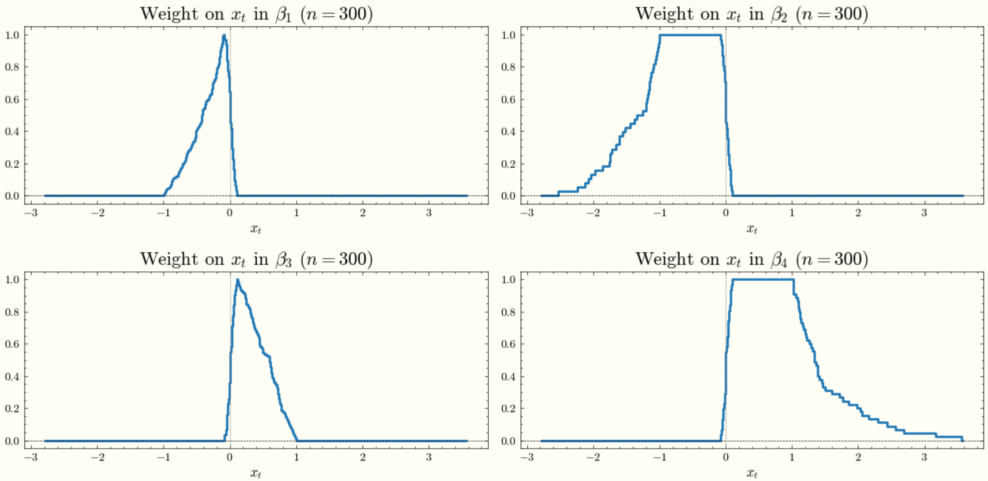
Instead of trying to measure effect of one type of shock relative to another..

..we can try to estimate effects on each region separately



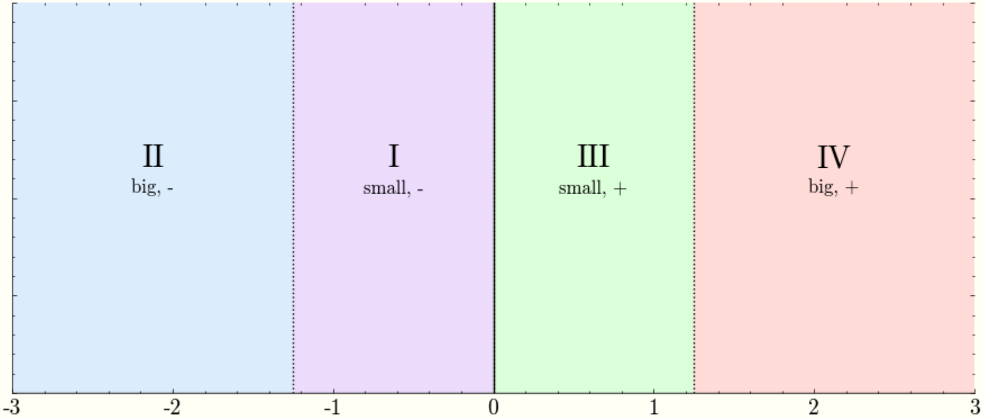
Indicator Functions Work Well

$$Y_{t+h} = \alpha + \beta_1 f_1(X_t) + \beta_2 f_2(X_t) + \beta_3 f_3(X_t) + \beta_4 f_4(X_t) + \gamma' \mathbf{W_t} + e_{t+h}$$



What Kind of Indicators?

1. Divide into 4 regions (robustness check: try others ex post)



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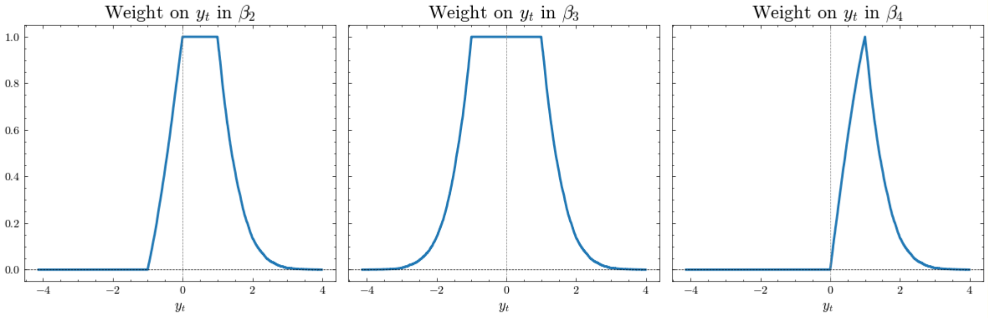

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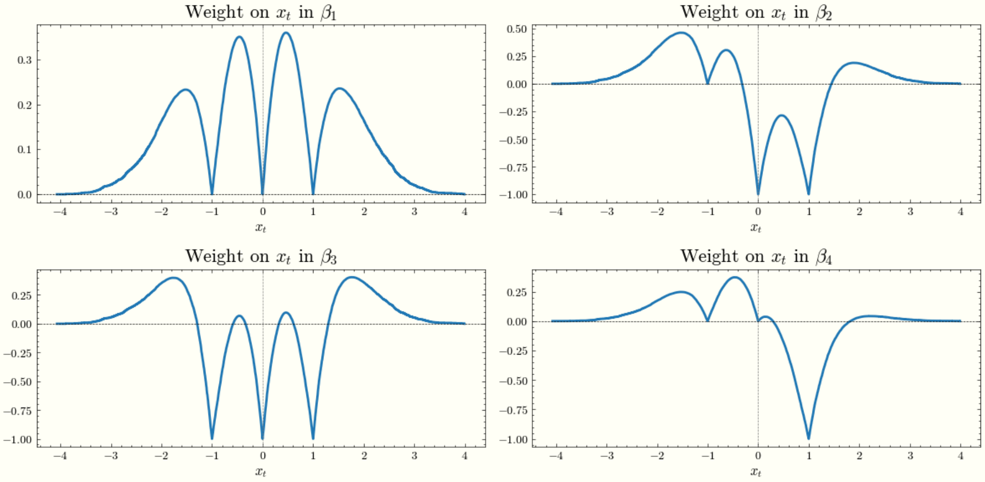
5. Rescale them by coefficient from projecting on X_t on $\{f_i\}$ (comprability)
   - (Delta method adjustment usually negligible)

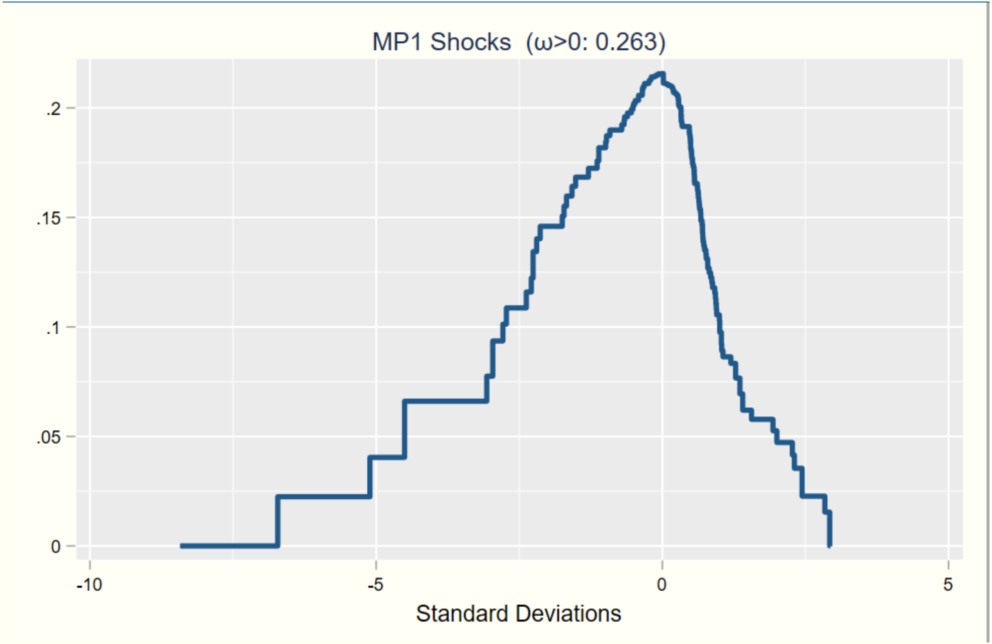
Final Words of Caution

1. Don't include X_t on its own (or interact it) and don't exclude anything other than 0



2. Think about what X_t is really measuring (Brennan et al., 2024)

MP1 Shocks (Monthly, 1988-2019)



Application: Monetary Policy Shocks (Monthly, 1988-2019)

Main questions:

1. Size Effects (are big and small the same)

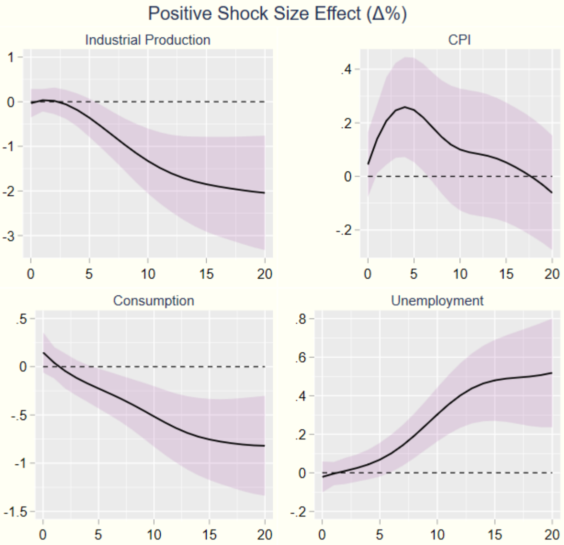
$$\beta_{\text{big}} = \beta_{\text{small}}$$

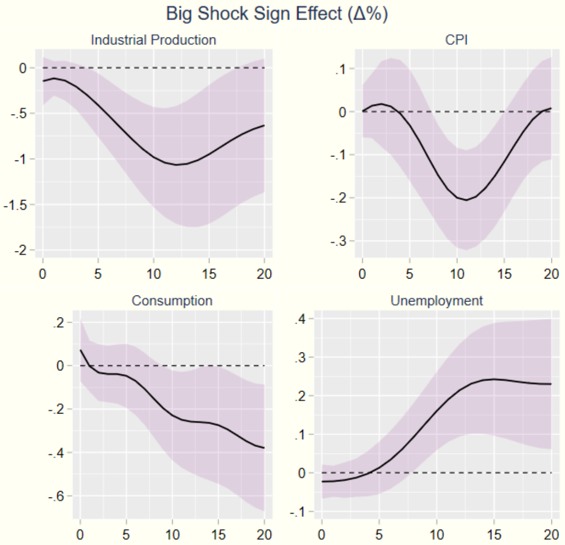
2. Sign Effects (are positive and negative asymmetric)

$$\beta_{\text{positive}} = -\beta_{\text{negative}}$$

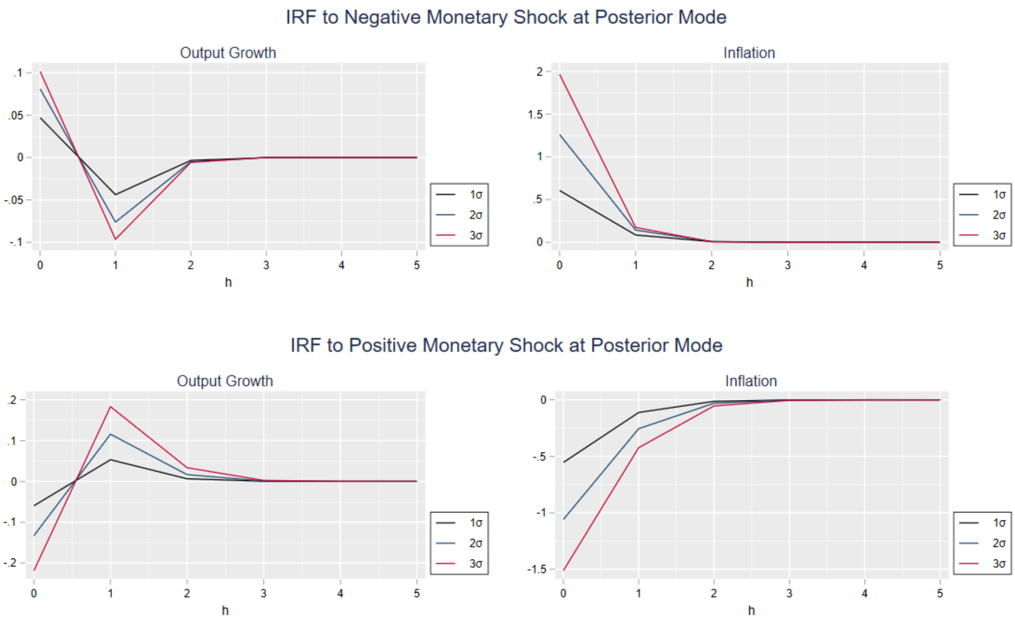
Can visualize this like a standard LP

Nonlinearities in MP Transmission





What could be the cause? (Aruoba et al., 2017)



Conclusion

- A peak inside the ”black box” (Goulet Coulombe and Klieber, 2024)

- Very easy to go wrong

- Partitioning works better than relative effects

- Ideally would want to extend to state dependence (Gonçalves et al., 2024)

- Many implications from persistent nonlinearities